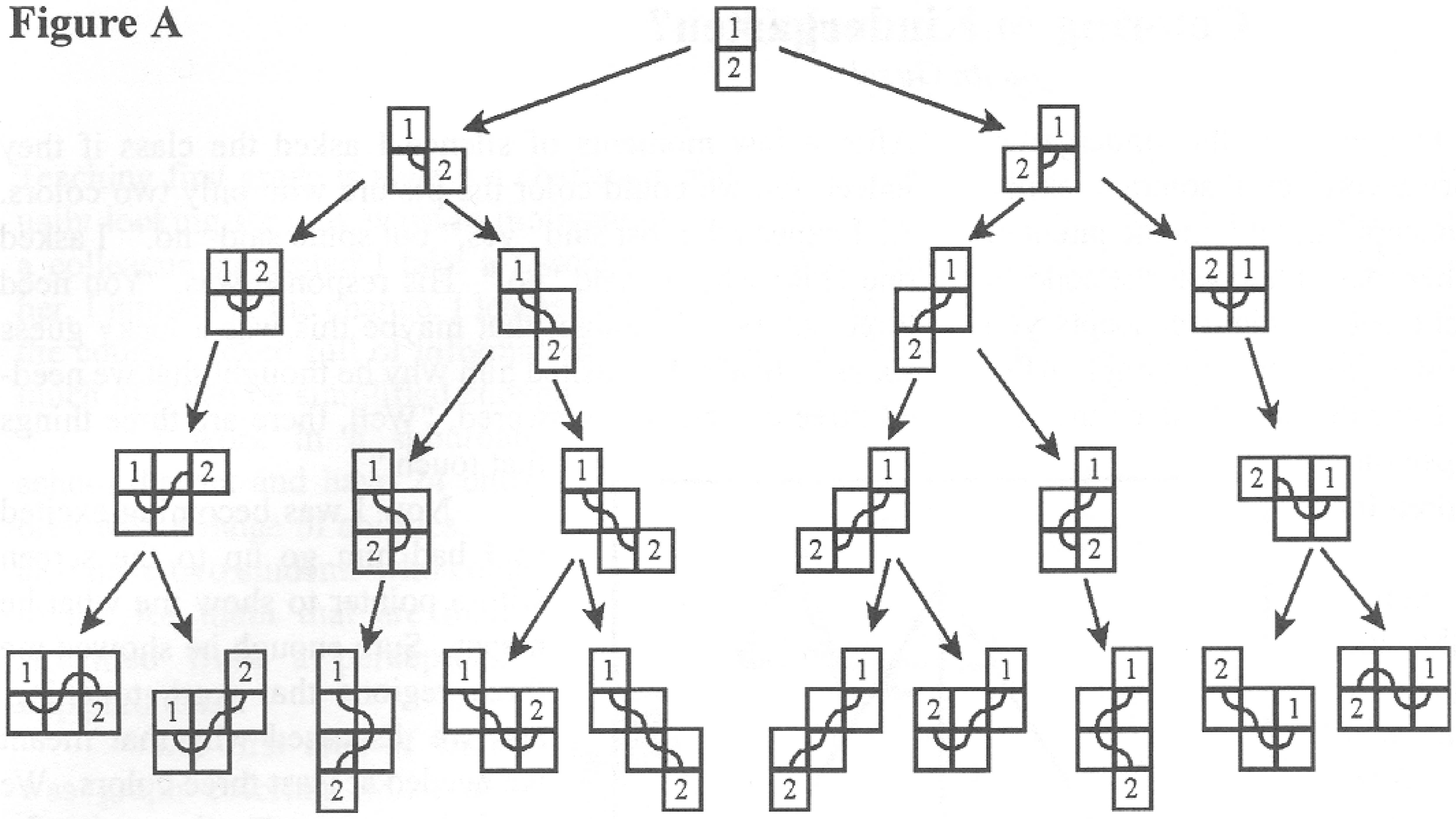
Figure A



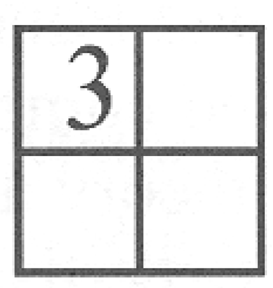
(continued from page 5)
choices for which piece comes second. This yields a total of 4×3=12 ways to select the pieces. For three intermediate pieces you have 4×3×2=24 ways, and for four intermediate pieces, there are 4×3×2×1=24 ways. Figure B combines this information with the number of track layouts to get the total number.

Number of Pieces	Number of Track Layouts	Middle Piece Arrangements	Total
2	1	1	1
3	2	4	8
4	4	12	48
5	6	24	144
6	10	24	240

Figure B

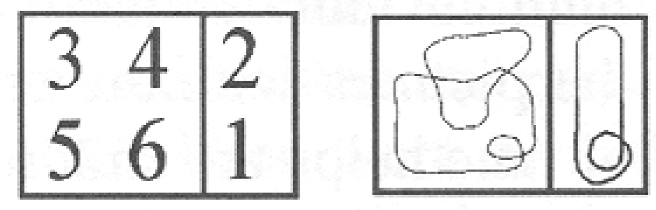
Thus there are 1+8+48+144+240=441 different track layouts using pieces #1 and #2.

If you do not use pieces #1 and #2, you can put the other four pieces together to form a square. Consider choosing piece #3 for the upper-left corner (and note that there is only one orientation for this piece which will yield a continuous track): There are 3×2×1 = 6 ways to put the other three pieces into the square. For each of these, there is only one rotation of each piece which will yield a "legal" track, so this square shape yields 6 new arrangements. Thus the total number of tracks, in which the train can travel over each square in the arrangement, is 441+6=447.



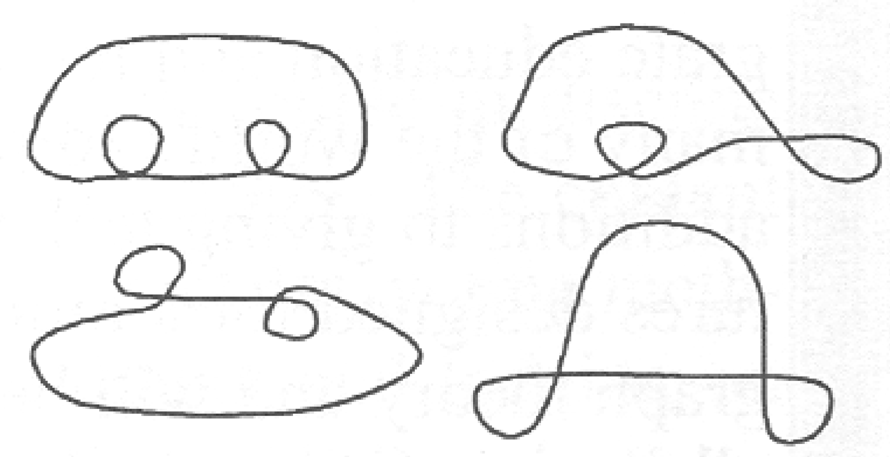
It is quite possible that the only layouts that the puzzle makers would consider acceptable are those in which the six pieces form a rectangle. In that case, there are two layouts — at the extremes of the bottom row of Figure A — and each can be realized in 24 ways, for a total of 48 layouts — not quite the "over 50" that they advertised.

If we change the rules to our game just slightly, the answer will be larger than the 447 found above. We required that the train must be able to reach each square in the layout. What if we drop that condition? The track pieces would be locked together but the train would not be able to cross each piece. An example is shown below.

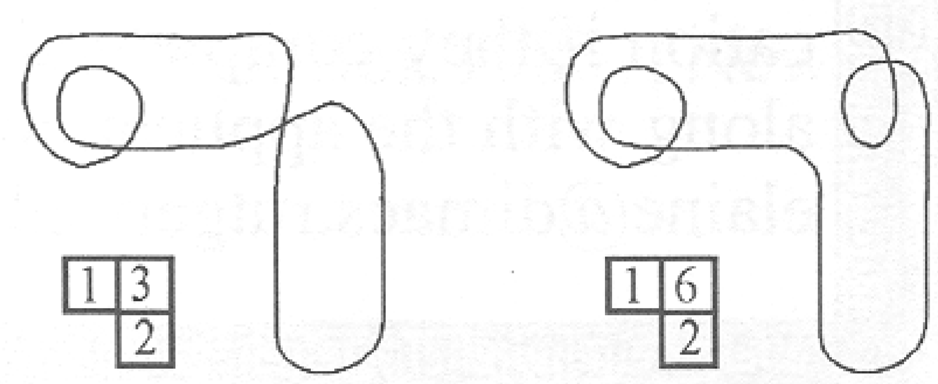


In addition to the 447 ways already found, we have those arrangements, as in the example here, which are formed by placing a #1-#2 unit along a side of one of the 6 square patterns. There are 4 sides to the square, and the #1-#2 unit can be placed two different ways on each side of the square, so there are 6×4×2= 48 new such ways. So with these relaxed rules we have 447+48=495 ways to assemble the track pieces.

On the other hand, if you want to define "different layouts" to mean "*essentially different* graphs which are circuits," then it becomes a totally different question. For example, we can define *vertices* to be places where two tracks cross and *loops* as track sections that begin and end at the same intersection without going through another intersection. We can then say that in order for two layouts to be different, they cannot have the same number of vertices which are connected in the same way and which have the same number of loops at each vertex. For example, these four graphs are essentially the same:



With the definitions above, one can count the essentially different circuits that can be built using the tracks. For example, if you build a circuit using pieces #1-#3-#2 and another using #1-#6-#2, they are each made up of



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